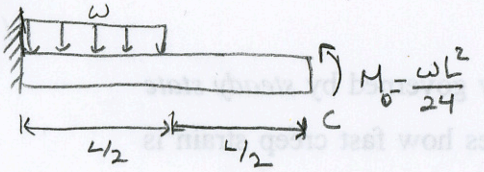
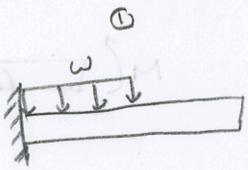
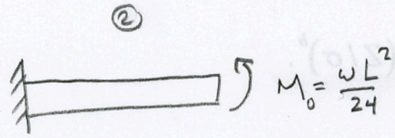
15.34 | Determine the slope and deflection at point C. $\begin{pmatrix} u'(L) = ? \\ u(L) = ? \end{pmatrix}$



$$\boxed{\begin{aligned} u(L) &= u_{1}(L) + u_{2}(L) \\ u'(L) &= u'_{1}(L) + u'_{2}(L) \end{aligned}}$$

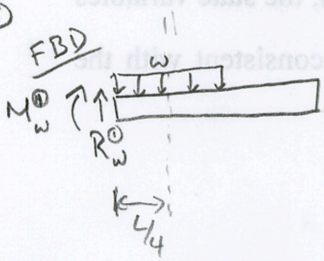
Find by superposition

= ① 

\+ ② 

①

FBD

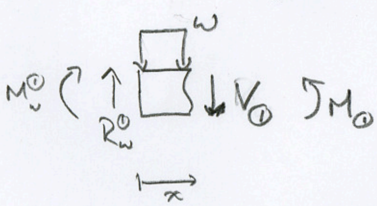


$$\Sigma F_y = 0 \Rightarrow R_w = w\left(\frac{L}{2}\right)$$

$$\Sigma M_{/wall} = 0 \Rightarrow M_w^{1} = \underbrace{-w\left(\frac{L}{2}\right)}_{\text{total force distributed}}\left(\frac{L}{4}\right) = -w\frac{L^2}{8}$$

effective moment arm

for $0 < x < \frac{L}{2}$



$$\Sigma M_{/cut} = 0$$

$$\Rightarrow M_1 = M_w^{1} + R_w^{1} x - \int_0^x w x \, dx$$

$$= -\frac{wL^2}{8} + \frac{wL}{2}x - \frac{w}{2}x^2$$

$$M_1(x) = EIu'' = -\frac{wL^2}{8} + \frac{wL}{2}x - \frac{w}{2}x^2$$

$$EIu' = -\frac{wL^2}{8}x + \frac{wL}{4}x^2 - \frac{w}{6}x^3 + \cancel{C_1}^{\,0}$$

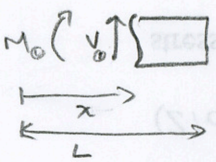
slope is zero at wall ($u'(0) = 0$)

$$EIu = -\frac{wL^2}{16}x^2 + \frac{wL}{12}x^3 - \frac{w}{24}x^4 + \cancel{C_2}^{\,0}$$

deflection is zero at wall ($u(0) = 0$)

15.34 cont'd

for $\frac{L}{2} < x < L$



$$\sum M_{/cut} = 0$$

$$\Rightarrow M_{①} = 0$$

$$EI\,u_{①}'' = 0$$

$$EI\,u_{①}' = C_1$$

$$EI\,u_{①} = C_1 x + C_2$$

Beam is continuous at $x = \frac{L}{2}$

$(*)$ $u_{①}\left(\frac{L}{2}^+\right) = u\left(\frac{L}{2}^-\right)$

$(**)$ $u_{①}'\left(\frac{L}{2}^+\right) = u'\left(\frac{L}{2}^-\right)$

$(**)$
$$C_1 = -\frac{wL^2}{8}\left(\frac{L}{2}\right) + \frac{wL}{4}\left(\frac{L}{2}\right)^2 - \frac{w}{6}\left(\frac{L}{2}\right)^3$$

$$= -\frac{wL^3}{16} + \frac{wL^3}{16} - \frac{wL^3}{48}$$

$$C_1 = -\frac{wL^3}{48}$$

$(*)$
$$-\frac{wL^3}{48}\left(\frac{L}{2}\right) + C_2 = -\frac{wL^2}{16}\left(\frac{L}{2}\right)^2 + \frac{wL}{12}\left(\frac{L}{2}\right)^3 - \frac{w}{24}\left(\frac{L}{2}\right)^4$$

$$= -\frac{wL^4}{64} + \frac{wL^4}{96} - \frac{wL^4}{384}$$

$$C_2 = \frac{wL^4}{384}$$

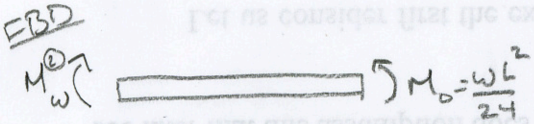
$$u_{①}' = \begin{cases} \dfrac{w}{24EI}\left(-3L^2 x + 6wLx^2 - 4wx^3\right) & \text{for } 0 < x < \frac{L}{2} \\[2ex] -\dfrac{wL^3}{48EI} & \text{for } \frac{L}{2} < x < L \end{cases}$$

15.34 cont'd

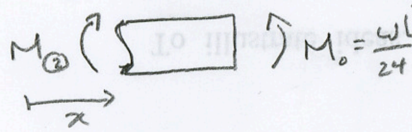
$$u_{①} = \begin{cases} \frac{w}{48EI}\left(-3L^2x^2 + 4wLx^3 - 2wx^4\right) & \text{for } 0 < x < \frac{L}{2} \\ \frac{wL^3}{384EI}\left(-8x + L\right) & \text{for } \frac{L}{2} < x < L \end{cases}$$

$$\boxed{u_{①}'(L) = -\frac{wL^3}{48EI}} \qquad \boxed{u_{①}(L) = \frac{-7wL^4}{384EI}}$$

FBD

② 

$$\Sigma M_{/cut} = 0 \Rightarrow M_{②} = \frac{wL^2}{24}$$



$$EI\,u_{②}'' = \frac{wL^2}{24}$$

$$EI\,u_{②}' = \frac{wL^2}{24}x + \cancel{C_1}^{\,0} \leftarrow u'(0) = 0$$

$$EI\,u_{②} = \frac{wL^2}{48}x^2 + \cancel{C_2}^{\,0} \leftarrow u(0) = 0$$

$$u_{②}' = \frac{wL^2}{24EI}x \qquad u_{②} = \frac{wL^2}{48EI}x^2$$

$$\boxed{u_{②}'(L) = \frac{wL^3}{24EI}} \qquad \boxed{u_{②}(L) = \frac{wL^4}{48EI}}$$

$$u'(L) = \frac{-wL^3}{48EI} + \frac{wL^3}{24EI} \qquad u(L) = \frac{-7wL^4}{384EI} + \frac{wL^4}{48EI}$$

$$\boxed{u'(L) = \frac{wL^3}{48EI}} \qquad \boxed{u(L) = \frac{wL^4}{384EI}}$$